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InIMC - 2017
India International Mathematics Competition
Organized by - CITY MONTESSORI INTER COLLEGE, RDSO Campus, Lucknow - INDIA
Phone: +91-522-2453546, E-mail: inimc@cmseducation.org, rdso@cmseducation.org

Invitational World Youth Mathematics Intercity Competition

Team Contest

Time limit: 70 minutes

Information:

- You are allowed 70 minutes for this paper, consisting of 10 questions printed on separate sheets. For questions 1, 3, 5, 7 and 9, only numerical answers are required. For questions 2, 4, 6, 8 and 10, full solutions are required.
- Each question is worth 40 points. For odd-numbered questions, no partial credits are given. There are no penalties for incorrect answers, but you must not give more than the number of answers being asked for. For questions asking for several answers, full credit will only be given if all correct answers are found. For even-numbered questions, partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on every question sheet.
- Enter your answers in the space provided after the individual questions on the question paper.
- During the first 10 minutes, the four team members examine the first 8 questions together, and altogether discuss them. Then they distribute the questions among themselves, with each team member is allotted at least 1 question.
- During the next 35 minutes, the four team members write down the solutions of their allotted problems on the respective question sheets, with no further communication / discussion among themselves.
- During the last 25 minutes, the four team members work together to write down the solutions of the last 2 questions on the respective questions sheets.
- You may not use instruments such as protractors, calculators and electronic devices.
- At the end of the contest, you must hand in the envelope containing all question sheets and all scratch papers.

English Version

Team: _____ ***Score:*** _____

For Juries Use Only

No.	1	2	3	4	5	6	7	8	9	10	Total	Sign by Jury
Score												
Score												

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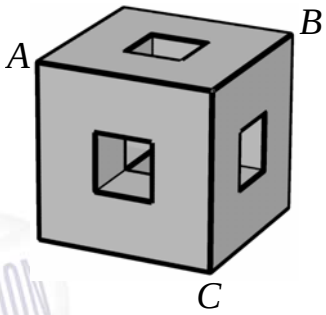
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TEAM CONTEST

28th July, 2017, Lucknow, India

Team：_____ Score：_____

1. The Void Cube is a $3\times3\times3$ cube with no center pieces just holes which can be looked through, as shown in the diagram. Consider the cross-section of the cube passing through the vertices *A*, *B* and *C*. Draw a picture of this cross section. (Shade the solid part of the triangle *ABC* in black and the hollow part in white.)



Answer: _____

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Team：_____ Score：_____

2. There are 314 coins in 21 open boxes. In each move, you can take 1 coin from each of any two boxes and put them into a third box. In the final move, you take all the coins from one box. What is the maximum number of coins you can get?

Answer: _____ coins

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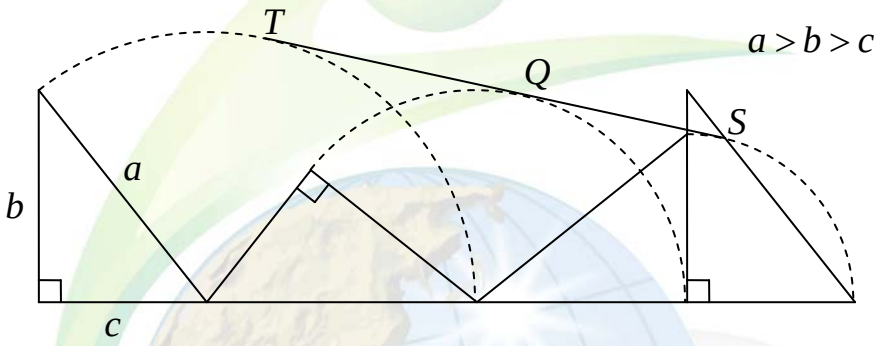
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TEAM CONTEST

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Team : _____ Score : _____

3. A right triangle with hypotenuse a stands on its shorter side $c=1$. We rotate it sequentially three times as it is shown below.



If TS is a common tangent of the three arcs, find a.

Answer: _____

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4. Find all ordered pairs (x, y) of positive integers which satisfy the equation $x^3 + y^3 = x^2 + 18xy + y^2$.

Answer: _____

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Team : _____ Score : _____

5. Let $P(x)$ be a polynomial of degree 3 and x_1, x_2, x_3 are the solutions of $P(x)=0$. Let $\dfrac{P\left(\frac{1}{3}\right)-P\left(\frac{-1}{3}\right)}{P(0)}=8$, $\dfrac{P\left(\frac{1}{4}\right)-P\left(\frac{-1}{4}\right)}{P(0)}=9$ and $x_1+x_2+x_3=35$.

Find the value of $\dfrac{x_2+x_3}{x_1}+\dfrac{x_1+x_3}{x_2}+\dfrac{x_1+x_2}{x_3}$.

Answer: _____

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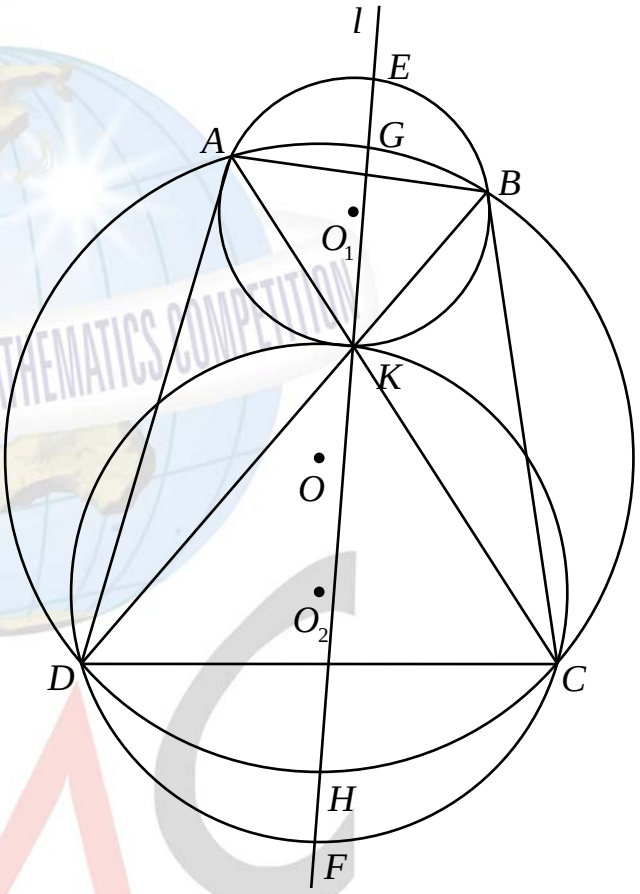
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Team：_____ Score：_____

6. The quadrilateral $ABCD$ is inscribed in a circle with center O. Connect AC and BD intersecting at K. O_1 is the circumcenter of triangle ABK and O_2 is the circumcenter of triangle CDK. A line l through K intersect the two circumcircles at E and F respectively, and the circumcircle of $ABCD$ at G and point H. Prove that $EG = FH$.



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7. Given that $a>0$, $x_n=a^n-\frac{1}{a^n}$ where $n=1, 2, 3, \ldots$. If $x_1=3$, find the units digit of x_{2017}.

Answer: _____

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Team：_____ Score：_____

8. Suppose x, y and z are all non-negative real numbers. Let k and m be the minimum possible values of $\dfrac{x^2+y^2+z^2+1}{xy+yz+z}$ and $\dfrac{x^2+y^2+z^2+1}{xy+y+z}$ respectively. Find $km+k+m$.

Answer: _____

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TEAM CONTEST

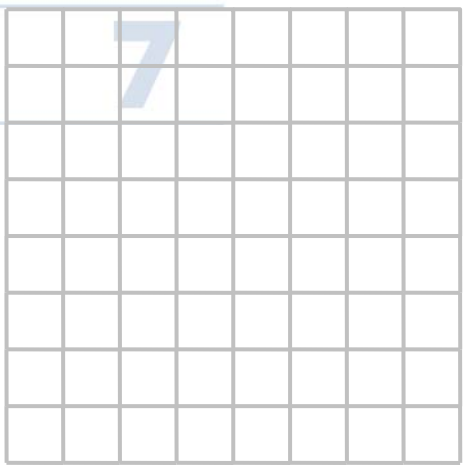
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Team : _____ Score : _____

9. Use three non-overlapping copies of a L-tetromino to construct a symmetric figure. Each L-tetromino must have a common point with at least one other L-tetromino.



Find a solution in which the copies may be rotated but not reflected.



Answer: _____

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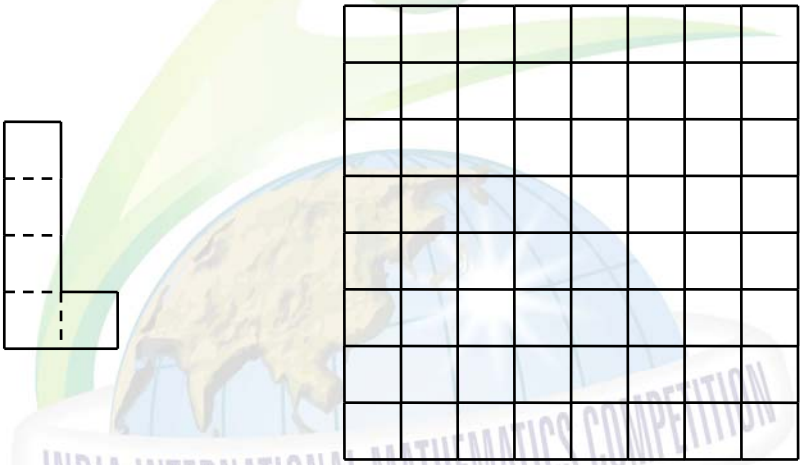
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Team：_____ Score：_____

10. How many ways are there of placing a single L-pentomino on the 8×8 chessboard so that it completely covers some five small squares of the chessboard?



Answer: _____ ways